A sub-wavelength holes diffraction radiation array

Shenggang Liu, Min Hu, Yaxin Zhang, Yuebao Li and Renbin Zhong
Terahertz Research Center, University of Electronic Science and Technology of China, Chengdu 610054, China,

***Abstract*—The letter explores the physics of the diffraction radiation of a subwavelength holes array. We show when a line current is uniformly moving above the array, a novel diffraction radiation array is discovered, in which each hole emits its own radiation in both sides of the array as a radiation cell, and all these cells together organize a distinct diffraction radiation array. Therefore, under the excitation of a line current, the subwavelength holes array becomes a radiation array. Making use of the integral equation method, analytical theory for such a novel diffraction radiation is built up. The results of the calculations of this theory agree with that by the computer simulation. The discovery and the theory of the novel diffraction radiation array are of great significance of the subwavelength holes array physics and optics.**

I. Introduction And Background

Subwavelength Holes Array (SHA) becomes a most attractive research area in modern science and technology for it is of interest to a wide spectrum of scientists, ranging from physicists, opticists and material scientists because of its very unusual characteristics and potential applications [1-2]. Recently, based on Subwavelength holes array, mimicking surface plasmons modes (MSPs) and their potential applications were presented and experimentally verified.

This letter explores the physics of the diffraction radiation of a subwavelength holes array. We show that when a line current of the electron beam is uniformly moving closely above the surface of a SHA, numerous physical phenomena occur, and these phenomena connected each other to form an entire physical process of the SHA diffraction system. Under the excitation of a line current the SHA becomes a diffraction radiation array in which each hole serves as a radiation cell, and all these cells form a radiation array, a very interesting, important and complicated physical process happens.

II. Computer Simulation

The schematic structure of the SHA and the results of the computer simulation are shown in Fig.1. It has been found that a line current passing through the surface of the array in one side excites the diffraction radiation in the both sides of the SHA with the same diffraction radiation angle, a little amplitude decay and a little phase delay as shown in Fig.1. Fig.2 (a) and (c) show that in the SHA each hole radiates its own radiation as a radiation cell, and all these cells together organize the entire diffraction radiation to form a diffraction radiation array. Fig.2 (a) and Fig.2 (c) also confirm the electromagnetic transmission through the subwavelength holes. A novel diffraction radiation inhibits distinct radiation characteristics and processes from others by involving the diffraction radiation in both upper and lower half spaces of the both sides of the SHA, and the electromagnetic transmission through the holes, etc. The above results show that under the line current excitation, the SHA becomes a diffraction radiation array.

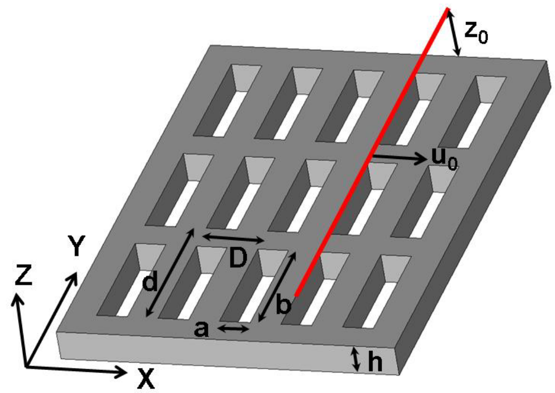

Fig.1 Schematic structure of the SHA

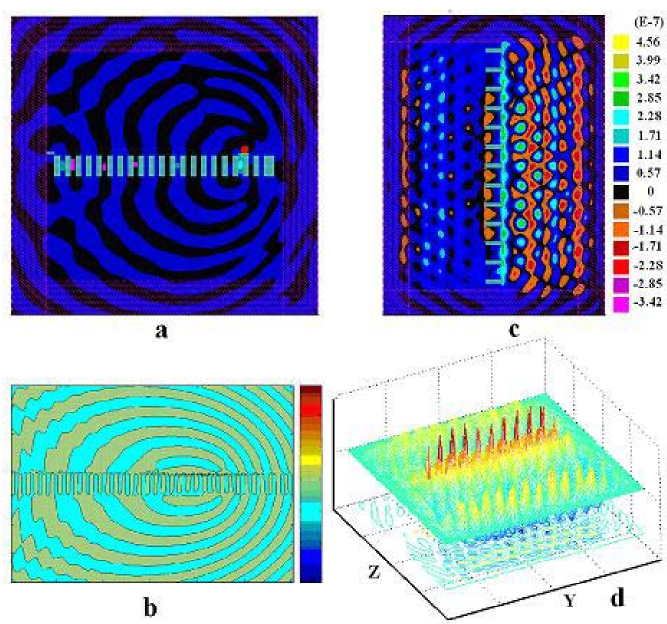

Fig.2 Contour maps: (a) Observed contour map of Bz on x-y plane (b) Calculated contour map of Ex (c) Observed contour map of Bz (d) Observed 3D contour map of Bz field

III. Theory

The physical processes of the novel diffraction radiation system can be described as below. Based on the Bloch-Floquet theorem, a set of evanescent waves induced by line current should be expanded into space harmonics to meet the boundary conditions. Some of these space harmonics propagate along the

978-1-4244-2120-6/08/$25.00 ©IEEE.

surface of the SHA as surface waves, while a part of them is diffracted in the upper half space taking the energy away. The propagating surface waves transmit through the holes of the array and propagate along the surface of the back side of the SHA, and part of them forms the diffraction radiation in the lower half space.

Making use of the integral equations with relevant Green' functions the analytical theory has been established. The modal has been divided into three regions: upper half space, lower half space and the holes. The proper Green's functions of these regions should be found and used to solve the diffraction and transmission fields separately. Finally, the boundary conditions are matched to determine the coefficients of the wave fields. The incident wave fields can be found by solving the Helmholtz equation. Making use of the Residue theory and keep only one of the two residues, the solution of Ex can be obtained.

$$E_x^i(x,y,z;\omega) = -j\frac{\mu_0\omega q}{2k_z^i}\mathrm{e}^{jk_x x - k_z^i|z-z_0|}$$

where $k_x = \omega/u_0, k_z^i = \sqrt{k_0^2 - k_x^2}$

To obtain the diffraction radiation wave field Ed and transmission field E^t, the following type integral equations should be solved.

$$E^{d,t}(x,y,z;\omega) = \oiint\left(G\nabla E^{d,t} - E^{d,t}\nabla G\right)\cdot ds$$

where Ed is the radiation wave field and $G = G(x,y,z;x',y',z')$ is the required three dimensional Green's function, and $\nabla G, \nabla E^d$ are the gradient of the Green's function and the electric field in the outer normal direction, respectively.

Now the following three Green's functions have been solved for the integral equations of the diffraction radiation in upper and lower half spaces and the transmission wave into one hole:

$$G^{ra,rb} = \sum_m \sum_n \frac{\dfrac{jabQ}{2k_{znm}dD}\left(1-e^{jk_{xm}D}\right)\sin c\left(\dfrac{k_{xm}a}{2}\right)e^{jk_{xm}(x-x')}e^{jk_{zmn}(z-z')}}{\cos\left(\dfrac{2n\pi}{d}y\right)\cos\left(\dfrac{2n\pi}{d}y'\right)}$$

$$G^p = \frac{2j}{abk_{zh}}\cos\left(\frac{\pi}{b}y\right)\cos\left(\frac{\pi}{b}y'\right)\left(e^{jk_{zh}(z-z')} + e^{-jk_{zh}(z-z')}\right)$$

G^{ra}, G^p and G^{rb} are the relevant Green's function, respectively and the parameters are shown as follow,

$$k_{xm} = k_x + \frac{2m\pi}{D}, k_{zmn}^2 = k_0^2 - k_{xm}^2 - \left(\frac{2n\pi}{d}\right)^2, k_{zh}^2 = k_0^2 - \left(\frac{\pi}{b}\right)^2$$

For the simplicity without sacrificing the generality, only the lowest mode in the waveguide, the TE_{10} mode is considered. Making use of the integration equation, the fields in three regions are obtained:

$$E^{ra}(x,y,z;\omega) = \sum_m \sum_n A_r B_1 e^{jk_{xm}x} e^{jk_{zmn}z}\cos\left(\frac{2n\pi}{d}y\right)$$

$$E^t(x,y,z;\omega) = \sum_m \sum_n A_p\left(B_2 e^{jk_{zh}z} + B_3 e^{-jk_{zh}z}\right)e^{jk_{xm}x}\cos\left(\frac{2n\pi}{d}y\right)$$

$$E^{rb}(x,y,z;\omega) = \sum_m \sum_n A_r B_4 e^{jk_{xm}x} e^{jk_{zmn}z}\cos\left(\frac{2n\pi}{d}y\right)$$

To analysis the characteristics of Surface Plasmon waves excited by line current, we develop the dispersion theory of the SHA. The theoretical analysis gives the dispersion equation for the eigenvalue approach of the Maxwell's equations with suitable boundary conditions. The fundamental mode TE_{10} is considered only because it is easiest to be excited in the holes. Using Floquet theory, the electric and magnetic fields and the dispersion equation of surface plasmon modes can be obtained:

$$\frac{k_{zh}}{k_0^2} = \frac{a}{D}\sum_{m=-\infty}^{\infty}\sin c^2\left(\frac{k_{xm}a}{2}\right)\frac{1}{k_{zmn}}$$

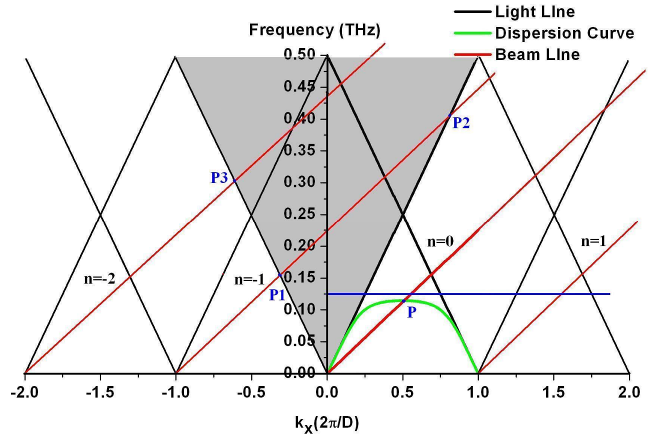

Fig.3 Dispersion diagram

The calculated dispersion curve (the green curve) of the SHA in the first apace harmonic region is shown in Fig.2. The work point is the cross of beam line (the red line) and dispersion curve below the cut-off frequency (the blue line) of holes. The space harmonics fall into the shadow region circled by the two light lines can radiate from the surface of the SHA. It shows that only the negative space harmonics can be the radiation wave. For instance, the first negative harmonic (n=-1) can radiate from P1 point to P2 and the second negative harmonic can radiate from P3.

IV. NUMERICAL CALCULATION

Fig.4 show the observed fields in fixed points in the upper and lower half spaces with the same distance to each surfaces of the SHA. It has been found that a line current passing throng the surface of the array in one side excites the diffraction radiation in the both sides of the SHA with the same diffraction radiation angle, a little amplitude decay and a little phase elay.

Fig.5 (a) shows the calculated radiation fields in the upper and lower half spaces of the both sides of the SHA, these calculations agree with that obtained by the computer simulation given in Fig.1. Fig.5 (b) shows the calculated frequencies of the diffraction radiation at a fixed point shown in Fig.1 (b) with red color. The line current is moving from left to the right, so the radiation angle is changing. The observed diffraction radiation frequencies at the fixed point are time dependence ranging from 0.175THz to 0.337THz shown in Fig.5 (a), and that range from 0.177THz to 0.389THz shown in Fig.5 (b) are the calculated frequencies disregarding finite boundary. Observed field E^p in the centre of the holes and the calculated results based on theory are the same frequency which equals 0.123THz agreed with the work point on the dispersion curve shown in Fig.2. All these calculated results are well agreed with that observed by computer simulation.

978-1-4244-2120-6/08/$25.00 ©IEEE.

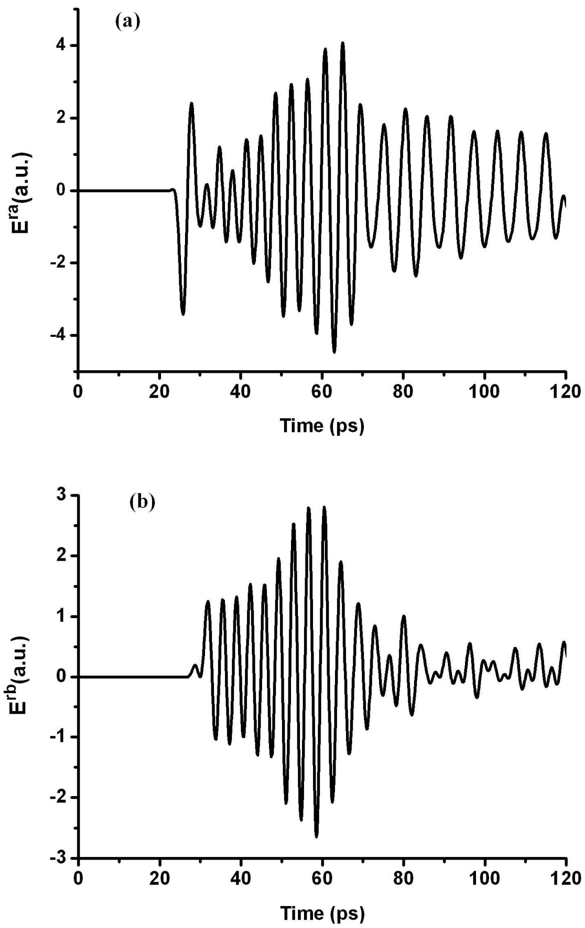

Fig.4 Observed Radiation fields in upper and lower half spaces

V. CONCLUSION

In conclusion, a distinct novel mechanism of diffraction radiation has been discovered, and the analytical theory has been established. A line current uniformly moving above the subwavelength array surface in one side excites diffraction radiation in the upper and lower half spaces of the both sides of the SHA with similar radiation angle, comparable amplitude and a little phase delay. It is a three dimensional SHA diffraction radiation in which each hole, serving as a radiation cell, emits its own radiation and the diffraction radiation of all these cells form a radiation array. That is under the line current excitation, the SHA becomes a diffraction radiation array.
The electromagnetic penetration behavior in the SHA is confirmed in the case of line current excitation. The penetrated wave below cutoff goes into the open entrance through the hole and then emits radiation from the lower open end. The diffraction radiation from all the holes in the back sides of the SHA form a radiation array again in the lower half space.
Making use of the integral equation, the theory of this novel SHA diffraction radiation array has been worked out. The results of the numerical calculations based on this theory for the contour map as well as for the radiation frequencies of radiation array all are agreed with that obtained by means of the computer simulation.

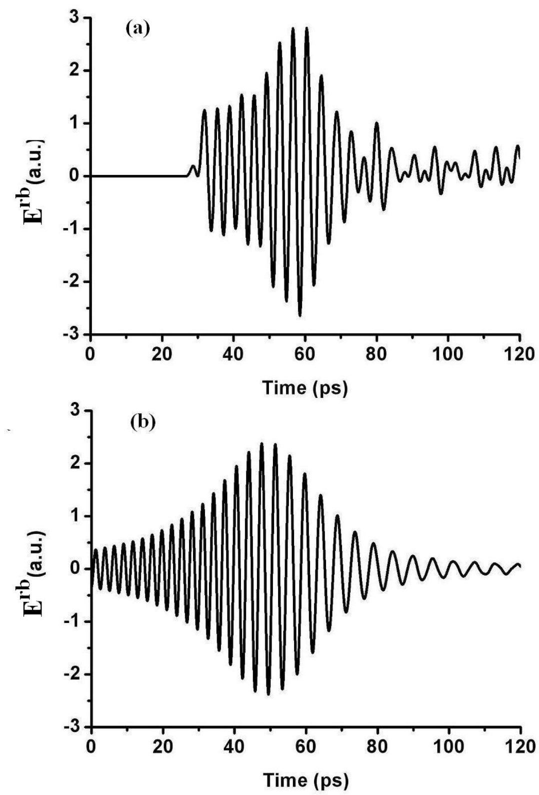

Fig.5 Observed and calculated Radiation fields The behavior of this novel diffraction radiation is very important for practical applications, in particular for the development of THz radiation sources and devices.
To summarize, the discovery and the theory of the novel diffraction radiation array are of great significance in physics and will greatly contribute to the sub-wavelength holes physics and optics.

ACKNOWLEDGEMENT

This work has been supported by National Key Program of Fundamental Research, No. 2007CB310401, and National Natural Science Foundation of China No.60472013.

REFERENCES

[1] J. B. Pendry et al, "Mimicking Surface Plasmons with Structured Surface" *Science*, vol. 305, pp 847-848, 2004.
[2] T. W. Ebbesen et al, "Extraordinary optical transmission through sub-wavelength hole arrays" Nature, vol. 391, pp. 667–669,1998.
[3] S.J. Smith and E. M. Purcell, "Visible light from localized surface charges moving across grating" Phys. Rev. 92 , pp1069,1953.
[4] P. M. Vandenberg, "Smith-purcell radiation from a line charge moving parallel to a reflection grating" J. Opt. Soc. Am 63, pp689-698,1973